Profit Analysis and Availability of a repairable redundant3-out-of-4system involving Preventive Maintenance

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Abstract: Studies on two unit repairable standby system dealing with availability and profit analysis involving preventive maintenance are numerous. However little attention is paid on the study of evaluation of reliability characteristics such as availability, busy period and profit function of multi component system such as 2-out-of-3 system involving four types of failures. The purpose of the present paper is to carry out the reliability of a redundant system with preventive maintenance. In this paper, we studied the availability and profit analysis of a repairable redundant 3-out-of-4 system with preventive maintenance involving four types of failures and develop explicit expressions for steady-state availability and profit function. The failure time distributions are exponential whereas the repair distribution is arbitrary or general. The expression for reliability is derived using supplementary variable technique. The numerical results for a particular case have also been made. Some particular cases have also been obtained analytically and graphically to see the impact of preventive maintenance on some system measures of effectiveness .Certain important result have been evaluated as special cases. Results have shown that system with preventive maintenance is better in terms of system effectiveness than system without preventive maintenance.

Index Terms: Availability, Profit, redundancy, preventive maintenance, busy period, supplementary variable technique.

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1 Introduction

Studies on redundant system are becoming more and richer day by day due to the fact that numbers of researchers in the field of reliability of redundant system are making huge contributions. Models of redundant systems as well as methods of evaluating system reliability indices such as mean time to system failure (MTSF), system availability, busy period of repairman, profit analysis, etc. have been studied in order to improve the system effectiveness.

There are systems of three/four units in which two/three units are sufficient to perform the entire function of the system. Examples of such systems are 2-out-of-3, 2-out-of-4, or 3-out-of-4 redundant systems. These systems have wide application in the real world. The communication system with three transmitters can be sited as a good example of 2-out-of-3 redundant system

Many research results have been reported on reliability of 2-out-of-3 redundant systems. For example, Chander and Bhardwaj [1], analyzed reliability models for 2-out-of-3 redundant system subject to conditional arrival time of the server. Chander and Bhardwaj [2] present reliability and economic analysis of 2-out-of-3 redundant system with priority to repair. Bhardwaj and Malik [3] studied MTSF and cost effectiveness of 2-out-of-3 cold standby system with probability of repair and inspection.

El-Said [5] and Haggag [6] examined the cost analysis of two unit cold standby system involving preventive maintenance respectively. Wang et al [4] examined the cost benefit analysis of series systems with cold standby components and repairable service station. Haggag [7] analyzed cost analysis of repairable k-out-of-n system with dependable failures and standby support. Wang and Kuo [8] studied the cost and probabilistic analysis of series system with mixed standby components. Wang et al [9] studied cost benefit analysis of series systems with warm standby components involving general repair time where the server is not subject to breakdowns. Ibrahim Yusuf [10] has studied availability and Profit Analysis of 3-out-of-4 Repairable System with Preventive Maintenance.

In this study, we developed the explicit expressions for availability and profit function and perform simulations to see the behavior of the system and comparison is perform through simulations. This study is an extension of the work of Yusuf [10]. The expression for reliability is derived using supplementary variable technique.

The aim of this study is to evaluate mean time to system failure (MTSF), system availability, steady state availability; busy period and profit function for the repairable redundant 3-out-of-4 system involving Preventive Maintenance by using supplementary variable technique. The result compare between the system (with and without) Preventive Maintenance theoretically and graphically for MTSF, availability, busy period and profit function. The results of this paper indicated that the better maintenance of parts of the system originated better reliability, performance of the system and reduced the total cost.

Notations:

$\alpha_i(x)$: General repair rates for unit i; i=1,2,3,4.

β_i : Constant failure rates for unit I; i=1,2,3,4

$\delta(y)$ **:** General rate end of preventive maintenance.

λ : Constant rate for taking a unit into preventive maintenance

O : The unit is operative.

S : The unit is standby.

O_P : The operative unit is under Preventive Maintenance.

S_P :The standby unit is under Preventive Maintenance.

F_{Ri} : The i^{th} failed unit is under repair.

F_{Wi} : The i^{th} failed unit is waited for repair.

$\bar{F}(s)$: Laplace transform of the function F(t)

$P_0(t)$: Probability that the system is in operable state S_0 at time t.

$P_i(t)$: Probability that the system is in state S_i at time t, i = 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 13.

$P_i(x,t)$: Probability that the system is in state S_i at time t, and under repair, elapsed repair time is x.

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$P_i(y,t)$: Probability that the system is in state S_i at time t, and under Preventive Maintenance, elapsed repair time is y.

According to Davis formula, there exists a relation between repair rate and the corresponding pdf, i.e.,

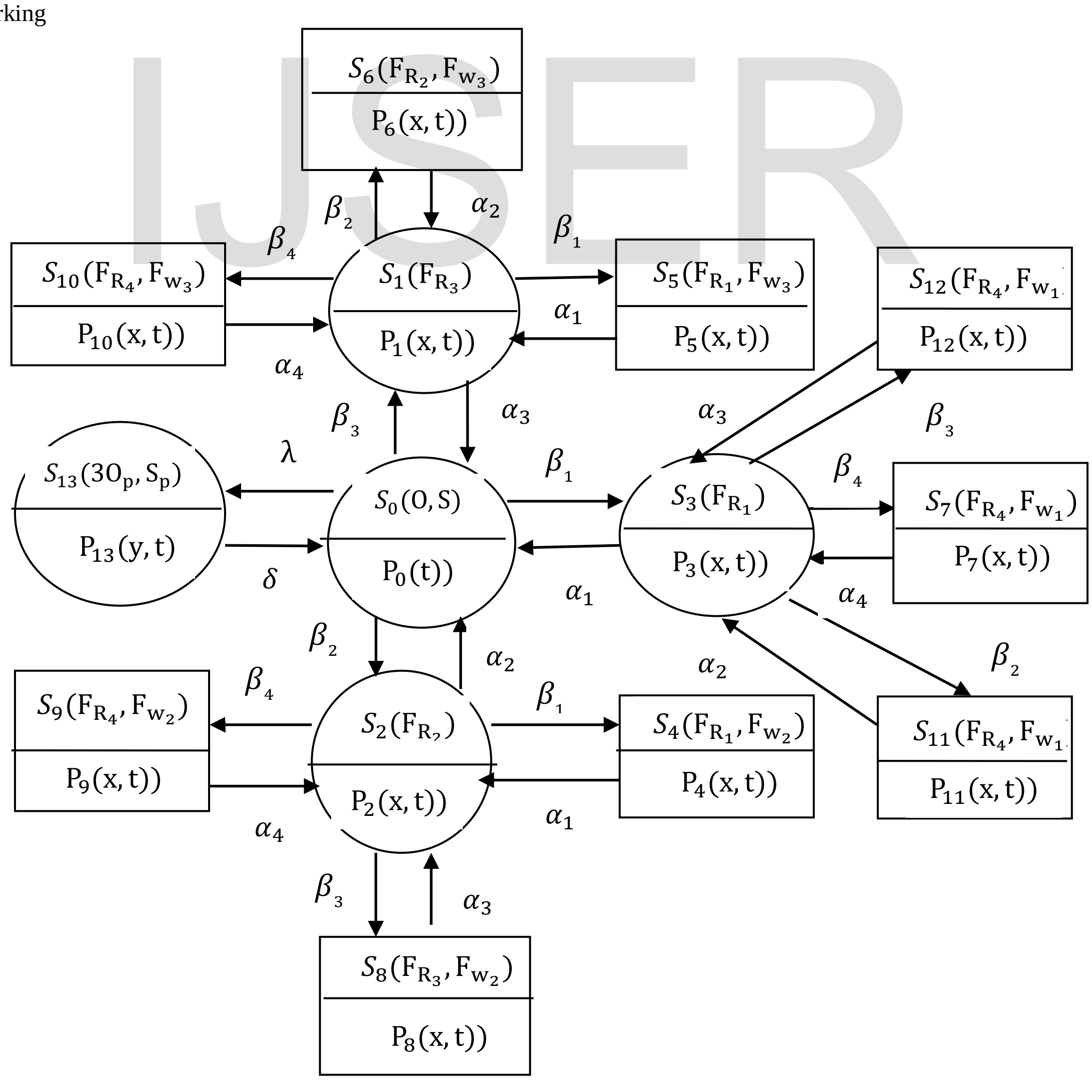
$$F(x) = \alpha_1(x)exp\left(-\int_0^x \alpha_1(x)dx\right)dx, \mathrm{G}(x) = \alpha_2(x)exp\left(-\int_0^x \alpha_2(x)dx\right)dx,$$

$$\mathrm{H}(x) = \alpha_3(x)exp\left(-\int_0^x \alpha_3(x)dx\right)dx, \mathrm{U}(x) = \alpha_4(x)exp\left(-\int_0^x \alpha_4(x)dx\right)dx, \mathrm{V}(y) = \delta(y)exp\left(-\int_0^y \delta(y)dy\right)dy$$

2 Markov Modeling of the System

In this section, the 3-out-of-4 redundant system is described. Using supplementary variable technique, system equations are obtained for the analysis of state probabilities. The system comprise of four units in which at least three units most be in operational for the system to work. Malfunctioning of two units lead the system go to down. The units can work consecutively or randomly as can be seen in the states of the system given below. The states of the system according Markov chain is shown in Fig. 1 below.

State 0: initial state, all the three units works, one unit in standby, and the system is working
State 1: units 1, 2, and 4 are working; unit 3 is down and under repair, and the system is working
State 2: units 1, 3 and 4 are working, unit 2 is down and under repair, and the system is working
State 3: units 2, 3 and 4 are working, unit 1 is down and under repair, and the system is working
State 4: units one is down, under repair, units 2 is down and waiting for repair, units 3 and 4 are good, and the system failed
State 5: unit 1 is down, under repair, unit 3 is down, waiting for repair, units 2 and 4 are good, and the system failed
State 6: unit 1 and 4 are good, unit 3 is down, and waiting for repair, unit 2 is down, under repair, and the system failed
State 7: unit 1 is down, waiting for repair, units 2 and 3 are good, unit 4 is down, under repair, and the system failed
State 8: units 1 and 4 are good, unit 2 is down, and waiting for repair, unit 3 is down, under repair, and the system failed
State 9: units 1 and 3 are good, unit 2 is down, and waiting for repair, unit 4 is down, under repair, and the system failed
State 10: units 1 and 2 are good, unit 3 is down, and waiting for repair, unit 4 is down, under repair, and the system failed
State 11: units two is down, under repair, units one is down and waiting for repair, units 3 and 4 are good, and the system failed
State 12: unit 3 is down, under repair, unit 1 is down, waiting for repair, units 2 and 4 are good, and the system failed
State 13: all the units are under preventive maintenance, and the system is working.



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Good stste: Failed state: under maintenance

Figure (1): schematic diagram of the system

3 Assumptions-

1- The system consists of four components/units.

2- Initially three units are in operable condition of full capacity and the forth is cold standby.

3- The system can be in Operative state, failed state or preventive maintenance

4- The system is failed when the number of working component goes down below 2.

5. The system suffer four types of failures

6- Failure time follows exponential distribution

7- Repair time and preventive maintenance time follow general distribution

8- Repair is as good as new (Perfect repair).

4 Formulation of mathematical model

Let$P_i(t)$ be the probability that the system is in state at time t. By elementary probability and continuity arguments the difference differential equations for the stochastic process of the system shown in Figure(1), which is continuous in time and discrete in space are shown in Equations (4 – 1) - (4 - 5):

$$\left(\frac{\partial}{\partial t} + \beta_1 + \beta_2 + \beta_3 + \lambda\right) P_0(t) = \int_0^\infty \alpha_1(x) P_3(x,t)\, dx + \int_0^\infty \alpha_2(x) P_2(x,t)\, dx + \int_0^\infty \alpha_3(x) P_1(x,t)\, dx + \int_0^\infty \delta(y) P_{13}(y,t) dy \quad (4\text{-}1)$$

$$\left(\frac{\partial}{\partial t} + \frac{\partial}{\partial x} + \beta_1 + \beta_2 + \beta_4 + \alpha_3(x)\right) P_1(x,t) = 0 \quad (4-2)$$

$$\left(\frac{\partial}{\partial t} + \frac{\partial}{\partial x} + \beta_1 + \beta_3 + \beta_4 + \alpha_2(x)\right) P_2(x,t) = 0 \quad (4-3)$$

$$\left(\frac{\partial}{\partial t} + \frac{\partial}{\partial x} + \beta_2 + \beta_3 + \beta_4 + \alpha_1(x)\right) P_3(x,t) = 0 \quad (4-4)$$

$$\left(\frac{\partial}{\partial t} + \frac{\partial}{\partial x} + \alpha_1(x)\right) P_4(x,t) = 0 \quad (4-5)$$

$$\left(\frac{\partial}{\partial t} + \frac{\partial}{\partial x} + \alpha_1(x)\right) P_5(x,t) = 0 \quad (4-6)$$

$$\left(\frac{\partial}{\partial t} + \frac{\partial}{\partial x} + \alpha_2(x)\right) P_6(x,t) = 0 \quad (4-7)$$

$$\left(\frac{\partial}{\partial t} + \frac{\partial}{\partial x} + \alpha_4(x)\right) P_7(x,t) = 0 \quad (4-8)$$

$$\left(\frac{\partial}{\partial t} + \frac{\partial}{\partial x} + \alpha_3(x)\right) P_8(x,t) = 0 \quad (4-9)$$

$$\left(\frac{\partial}{\partial t} + \frac{\partial}{\partial x} + \alpha_4(x)\right) P_9(x,t) = 0 \quad (4-10)$$

$$\left(\frac{\partial}{\partial t} + \frac{\partial}{\partial x} + \alpha_4(x)\right) P_{10}(x,t) = 0 \quad (4-11)$$

$$\left(\frac{\partial}{\partial t} + \frac{\partial}{\partial x} + \alpha_2(x)\right) P_{11}(x,t) = 0 \quad (4-12)$$

$$\left(\frac{\partial}{\partial t} + \frac{\partial}{\partial y} + \alpha_3(x)\right) P_{12}(x,t) = 0 \quad (4-13)$$

$$\left(\frac{\partial}{\partial t} + \frac{\partial}{\partial y} + \delta(y)\right) P_{13}(y,t) = 0 \quad (4-14)$$

Boundary and Initial conditions

$$P_1(0,t) = \beta_3 P_0(t) + \int_0^\infty \alpha_1(x) P_5(x,t) dx + \int_0^\infty \alpha_2(x) P_6(x,t) dx + \int_0^\infty \alpha_4(x) P_{10}(x,t) dx \quad (4-15)$$

$$P_2(0,t) = \beta_2 P_0(t) + \int_0^\infty \alpha_1(x) P_4(x,t) dx + \int_0^\infty \alpha_3(x) P_8(x,t) dx + \int_0^\infty \alpha_4(x) P_9(x,t) dx \quad (4-16)$$

$$P_3(0,t) = \beta_1 P_0(t) + \int_0^\infty \alpha_4(x) P_7(x,t) dx + \int_0^\infty \alpha_2(x) P_{11}(x,t) dx + \int_0^\infty \alpha_3(x) P_{12}(x,t) dx \quad (4-17)$$

$$P_4(0,t) = \beta_1 P_2(t) \quad (4-18)$$

$$P_5(0,t) = \beta_1 P_1(t) \quad (4-19)$$

$$P_6(0,t) = \beta_2 P_1(t) \quad (4-20)$$

$$P_7(0,t) = \beta_4 P_3(t) \quad (4-21)$$

$$P_8(0,t) = \beta_3 P_2(t) \quad (4-22)$$

$$P_9(0,t) = \beta_4 P_2(t) \quad (4-23)$$

$$P_{10}(0,t) = \beta_4 P_1(t) \quad (4-24)$$

$$P_{11}(0,t) = \beta_2 P_3(t) \quad (4-25)$$

$$P_{12}(0,t) = \beta_3 P_3(t) \quad (4-26)$$

$$P_{13}(0,t) = \lambda P_0(t) \quad (4-27)$$

$P_0(0) = 1$, $P_i(0) = 0$, i=1, 2, 3, 4,5,6,7,8,9,10,11, 12, 13. (4 – 28)

5 Solution of the Model

By taking Laplace transform of equations (4-1)-(4-27), and using initial condition (4 – 28), we get:

$$(s + \beta_1 + \beta_2 + \beta_3 + \lambda)\bar{P}_0(s) = 1 + \int_0^\infty \alpha_1(x)\bar{P}_3(x,s)\, dx + \int_0^\infty \alpha_2(x)\bar{P}_2(x,s)\, dx + \int_0^\infty \alpha_3(x)\bar{P}_1(x,s)\, dx + \int_0^\infty \delta(y)\bar{P}_{13}(y,s) dy \quad (5\text{-}1)$$

$$\left(\frac{\partial}{\partial x} + s + \beta_1 + \beta_2 + \beta_4 + \alpha_3(x)\right)\bar{P}_1(x,s) = 0 \quad (5-2)$$

$$\left(\frac{\partial}{\partial x} + s + \beta_1 + \beta_3 + \beta_4 + \alpha_2(x)\right)\bar{P}_2(x,s) = 0 \quad (5-3)$$

$$\left(\frac{\partial}{\partial x}+s+\beta_2+\beta_3+\beta_4+\alpha_1(x)\right)\bar{P}_3(x,s)=0 \quad (5-4)$$

$$\left(\frac{\partial}{\partial x}+s+\alpha_1(x)\right)\bar{P}_4(x,s)=0 \quad (5-5)$$

$$\left(\frac{\partial}{\partial x}+s+\alpha_1(x)\right)\bar{P}_5(x,s)=0 \quad (5-6)$$

$$\left(\frac{\partial}{\partial x}+s+\alpha_2(x)\right)\bar{P}_6(x,s)=0 \quad (5-7)$$

$$\left(\frac{\partial}{\partial x}+s+\alpha_4(x)\right)\bar{P}_7(x,s)=0 \quad (5-8)$$

$$\left(\frac{\partial}{\partial x}+s+\alpha_3(x)\right)\bar{P}_8(x,s)=0 \quad (5-9)$$

$$\left(\frac{\partial}{\partial x}+s+\alpha_4(x)\right)\bar{P}_9(x,s)=0 \quad (5-10)$$

$$\left(\frac{\partial}{\partial x}+s+\alpha_4(x)\right)\bar{P}_{10}(x,s)=0 \quad (5-11)$$

$$\left(\frac{\partial}{\partial x}+s+\alpha_2(x)\right)\bar{P}_{11}(x,s)=0 \quad (5-12)$$

$$\left(\frac{\partial}{\partial x}+s+\alpha_3(x)\right)\bar{P}_{12}(x,s)=0 \quad (5-13)$$

$$\left(\frac{\partial}{\partial y}+s+\delta(y)\right)\bar{P}_{13}(y,s)=0 \quad (5-14)$$

Boundary and Initial conditions

$$\bar{P}_1(0,s)=\beta_3\bar{P}_0(s)+\int_0^\infty\alpha_1(x)\bar{P}_5(x,s)dx+\int_0^\infty\alpha_2(x)\bar{P}_6(x,s)dx+\int_0^\infty\alpha_4(x)\bar{P}_{10}(x,s)dx \quad (5-15)$$

$$\bar{P}_2(0,s)=\beta_2\bar{P}_0(s)+\int_0^\infty\alpha_1(x)\bar{P}_4(x,s)dx+\int_0^\infty\alpha_3(x)\bar{P}_8(x,s)dx+\int_0^\infty\alpha_4(x)\bar{P}_9(x,s)dx \quad (5-16)$$

$$\bar{P}_3(0,s)=\beta_1\bar{P}_0(s)+\int_0^\infty\alpha_4(x)\bar{P}_7(x,s)dx+\int_0^\infty\alpha_2(x)\bar{P}_{11}(x,s)dx+\int_0^\infty\alpha_3(x)\bar{P}_{12}(x,s)dx \quad (5-17)$$

$$\bar{P}_4(0,s)=\beta_1\bar{P}_2(s) \quad (5-18)$$

$$\bar{P}_5(0,s)=\beta_1\bar{P}_1(s) \quad (5-19)$$

$$\bar{P}_6(0,s)=\beta_2\bar{P}_1(s) \quad (5-20)$$

$$\bar{P}_7(0,s)=\beta_4\bar{P}_3(s) \quad (5-21)$$

$$\bar{P}_8(0,s)=\beta_3\bar{P}_2(s) \quad (5-22)$$

$$\bar{P}_9(0,s)=\beta_4\bar{P}_2(s) \quad (5-23)$$

$$\bar{P}_{10}(0,s)=\beta_4\bar{P}_1(s) \quad (5-24)$$

$$\bar{P}_{11}(0,s)=\beta_2\bar{P}_3(s) \quad (5-25)$$

$$\bar{P}_{12}(0,s)=\beta_3\bar{P}_3(s) \quad (5-26)$$

$$\bar{P}_{13}(0,t)=\lambda\bar{P}_0(s) \quad (5-27)$$

Integrating equations (5 – 2)-(5 – 14)

$$\overline{P_1}(x,s)=\overline{P_1}(0,s)\exp[-(s+\beta_1+\beta_2+\beta_4)x-\int_0^x\alpha_3(x)dx] \quad (5-28)$$

$$\overline{P_2}(x,s)=\overline{P_2}(0,s)\exp[-(s+\beta_1+\beta_3+\beta_4)x-\int_0^x\alpha_2(x)dx \quad (5-29)$$

$$\overline{P_3}(x,s)=\overline{P_3}(0,s)\exp[-(s+\beta_2+\beta_3+\beta_4)x-\int_0^x\alpha_1(x)dx] \quad (5-30)$$

$$\bar{P}_4(x,s)=\bar{P}_4(0,s)\exp[-sx-\int_0^x\alpha_1(x)dx] \quad (5-31)$$

$$\overline{P_5}(x,s)=\overline{P_5}(0,s)\exp[-sx-\int_0^x\alpha_1(x)dx] \quad (5-32)$$

$$\overline{P_6}(x,s)=\overline{P_6}(0,s)\exp[-sx-\int_0^x\alpha_2(x)dx] \quad (5-33)$$

$$\overline{P_7}(x,s)=\overline{P_7}(0,s)\exp[-sx-\int_0^x\alpha_4(x)dx] \quad (5-34)$$

$$\overline{P_8}(x,s)=\overline{P_8}(0,s)\exp[-sx-\int_0^x\alpha_3(x)dx] \quad (5-35)$$

$$\overline{P_9}(x,s)=\overline{P_9}(0,s)\exp[-sx-\int_0^x\alpha_4(x)dx] \quad (5-36)$$

$$\overline{P_{10}}(x,s)=\overline{P_{10}}(0,s)\exp[-sx-\int_0^x\alpha_4(x)dx] \quad (5-37)$$

$$\overline{P_{11}}(x,s)=\overline{P_{11}}(0,s)\exp[-sx-\int_0^x\alpha_2(x)dx] \quad (5-38)$$

$$\overline{P_{12}}(x,s)=\overline{P_{12}}(0,s)\exp[-sx-\int_0^x\alpha_3(x)dx] \quad (5-39)$$

$$\overline{P_{13}}(y,s)=\overline{P_{13}}(0,s)\exp[-sx-\int_0^y\delta(y)dy] \quad (5-40)$$

Again integrating by parts equations (5 – 28)-(5 - 40) using equations (5 – 15)-(5 –27)

$$\overline{P_1}(s)=\int_0^\infty\overline{P_1}(x,s)\,dx$$
$$=\overline{P_1}(0,s)\left\{\int_0^\infty\exp\left[-(s+\beta_1+\beta_2+\beta_4)x-\int_0^x\alpha_3(x)dx\right]dx\right\}$$

$$=\overline{P_1}(0,s)\left\{\int_0^\infty\left[exp\left(-\int_0^x\alpha_3(x)dx\right)\right]d\left(\frac{\exp[-(s+\beta_1+\beta_2+\beta_4)x}{-(s+\beta_1+\beta_2+\beta_4)}\right)\right\}$$

$$=(s+\beta_1+\beta_2+\beta_4)^{-1}[1-\bar{\mathrm{H}}(s+\beta_1+\beta_2+\beta_4)]\overline{P_1}(0,s)$$

$$\therefore\overline{P_1}(s)=N_1(s)\overline{P_1}(0,s) \quad (5-41)$$

In the same manner we can get the other integrations:

$$\overline{P_2}(s)=N_2(s)\overline{P_2}(0,s) \quad (5-42)$$

$$\overline{P_3}(s)=N_3(s)\overline{P_3}(0,s) \quad (5-43)$$

$$\bar{P}_4(s)=N_4(s)\bar{P}_2(s) \quad (5-44)$$

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$\overline{P_5}(s) = N_5(s)\bar{P}_1(s)$ (5 – 45)

$\overline{P_6}(s) = N_6(s)\bar{P}_1(s)$ (5 – 46)

$\overline{P_7}(s) = N_7(s)\bar{P}_3(s)$ (5 – 47)

$\overline{P_8}(s) = N_8(s)\bar{P}_2(s)$ (5 – 48)

$\overline{P_9}(s) = N_9(s)\bar{P}_2(s)$ (5 – 49)

$\overline{P_{10}}(s) = N_{10}(s)\bar{P}_1(s)$ (5 – 50)

$\overline{P_{11}}(s) = N_{11}(s)\bar{P}_3(s)$ (5 – 51)

$\overline{P_{12}}(s) = N_{12}(s)\bar{P}_3(s)$ (5 – 52)

$\overline{P_{13}}(s) = N_{13}(s)\bar{P}_0(s)$ (5 – 53)

Where:

$N_1(s) = (s + \beta_1 + \beta_2 + \beta_4)^{-1}[1 - \bar{\mathrm{H}}(s + \beta_1 + \beta_2 + \beta_4)]$

$N_2(s) = (s + \beta_1 + \beta_3 + \beta_4)^{-1}[1 - \bar{G}(s + \beta_1 + \beta_3 + \beta_4)]$

$N_3(s) = (s + \beta_2 + \beta_3 + \beta_4)^{-1}[1 - \bar{F}(s + \beta_2 + \beta_3 + \beta_4)]$

$N_4(s) = \beta_1 s^{-1}[1 - \bar{F}(s)]$

$N_5(s) = \beta_1 s^{-1}[1 - \bar{F}(s)]$

$N_6(s) = \beta_2 s^{-1}[1 - \bar{G}(s)]$

$N_7(s) = \beta_4 s^{-1}[1 - \bar{U}(s)]$

$N_8(s) = \beta_3 s^{-1}[1 - \bar{H}(s)]$

$N_9(s) = \beta_4 s^{-1}[1 - \bar{U}(s)]$

$N_{10}(s) = \beta_4 s^{-1}[1 - \bar{U}(s)]$

$N_{11}(s) = \beta_2 s^{-1}[1 - \bar{G}(s)]$

$N_{12}(s) = \beta_3 s^{-1}[1 - \bar{H}(s)]$

$N_{13}(s) = \lambda s^{-1}[1 - \bar{V}(s)]$

And:

$$\bar{F}(s) = \int_0^\infty \exp(-sx)\,\alpha_1(x) exp\left(-\int_0^x \alpha_1(x)dx\right)dx,$$

$$\bar{G}(s) = \int_0^\infty \exp(-sx)\,\alpha_2(x) exp\left(-\int_0^x \alpha_2(x)dx\right)dx,$$

$$\bar{H}(s) = \int_0^\infty \exp(-sx)\,\alpha_3(x) exp\left(-\int_0^x \alpha_3(x)dx\right)dx,$$

$$\bar{U}(s) = \int_0^\infty \exp(-sx)\,\alpha_4(x) exp\left(-\int_0^x \alpha_4(x)dx\right)dx,$$

$$\bar{V}(s) = \int_0^\infty \exp(-sy)\,\delta(y) exp\left(-\int_0^y \delta(y)dx\right)dy$$

Also we have from equations (5 – 28)-(5 – 40) using equations (5 – 15)-(5 –27)

$\int_0^\infty \overline{P_1}(x,s)\,\alpha_3(x)dx = \overline{P_1}(0,s)\bar{\mathrm{H}}(s + \beta_1 + \beta_2 + \beta_4)$ (5 – 54)

$\int_0^\infty \overline{P_2}(x,s)\,\alpha_2(x)dx = \overline{P_2}(0,s)\bar{\mathrm{G}}(s + \beta_1 + \beta_3 + \beta_4)$ (5 – 55)

$\int_0^\infty \overline{P_3}(x,s)\,\alpha_1(x)dx = \overline{P_3}(0,s)\bar{\mathrm{F}}(s + \beta_2 + \beta_3 + \beta_4)$ (5 – 56)

$\int_0^\infty \overline{P_4}(x,s)\alpha_1(x)\,dx = \beta_1\bar{P}_2(s)\bar{\mathrm{F}}(s)$ (5 – 57)

$\int_0^\infty \overline{P_5}(x,s)\,\alpha_1(x)dx = \beta_1\bar{P}_1(s)\bar{\mathrm{F}}(s)$ (5 – 58)

$\int_0^\infty \overline{P_6}(x,s)\,\alpha_2(x)dx = \beta_2\bar{P}_1(s)\bar{\mathrm{G}}(s)$ (5 – 59)

$\int_0^\infty \overline{P_7}(x,s)\alpha_4(x)\,dx = \beta_4\bar{P}_3(s)\bar{\mathrm{U}}(s)$ (5 – 60)

$\int_0^\infty \overline{P_8}(x,s)\,\alpha_3(x)dx = \beta_3\bar{P}_2(s)\bar{\mathrm{H}}(s)$ (5 – 61)

$\int_0^\infty \overline{P_9}(x,s)\,\alpha_4(x)dx = \beta_4\bar{P}_2(s)\bar{\mathrm{U}}(s)$ (5 – 62)

$\int_0^\infty \overline{P_{10}}(x,s)\alpha_4(x)\,dx = \beta_4\bar{P}_1(s)\bar{\mathrm{U}}(s)$ (5 – 63)

$\int_0^\infty \overline{P_{11}}(x,s)\alpha_2(x)\,dx = \beta_2\bar{P}_3(s)(\bar{\mathrm{G}}(s))$ (5 – 64)

$\int_0^\infty \overline{P_{12}}(x,s)\,\alpha_3(x)dx = \beta_3\bar{P}_3(s)(\bar{\mathrm{H}}(s))$ (5 – 65)

$\int_0^\infty \overline{P_{13}}(x,s)\,\delta(y)dy = \lambda\overline{P_0}(s)\bar{\mathrm{V}}(s)$ (5 – 66)

Now from equation (5- 41) using equation (5-15), (5-58), (5-59), (5-63), we get:

$\overline{P_1}(s) = A_1(s)\overline{P_0}(s),$ (5 – 67)

Also, from equation (5- 42) using equation(5-16), (5-57), (5-61), (5-62), we get:

$\overline{P_2}(s) = A_2(s)\overline{P_0}(s),$ (5 – 68)

Also, from equation (5- 35) using equation(5-17), (5-60), (5-64), (5-65), we get:

$\overline{P_3}(s) = A_3(s)\overline{P_0}(s),$ (5 – 69)

Lastly, by using equations (5-54), (5-55), (5-56), (5-66) (Use also Eqs. 41,42, 43,38) in equation (5 – 1), we get:

$\overline{P_0}(s) = [s + \beta_1 + \beta_2 + \beta_3 + \lambda - N_1^{-1}(s)A_1(s)\bar{F}(s + \beta_1 + \beta_2 + \beta_4) - N_2^{-1}(s)A_2(s)\bar{G}(s + \beta_1 + \beta_3 + \beta_4) - N_3^{-1}(s)A_3(s)\bar{H}(s + \beta_2 + \beta_3 + \beta_4) - \lambda\bar{V}(s)]^{-1} = \frac{1}{A(s)}$ (5 – 70)

In similar way other probabilities are:

$\overline{P_4}(s) = A_4(s)\overline{P_0}(s)$ (5 – 71)

$\overline{P_5}(s) = A_5(s)\overline{P_0}(s)$ (5 – 72)

$\overline{P_6}(s) = A_6(s)\overline{P_0}(s)$ (5 – 73)

$\overline{P_7}(s) = A_7(s)\overline{P_0}(s)$ (5 – 74)

$\overline{P_8}(s) = A_8(s)\overline{P_0}(s)$ (5 – 75)

$\overline{P_9}(s) = A_9(s)\overline{P_0}(s)$ (5 – 76)

$\overline{P_{10}}(s) = A_{10}(s)\overline{P_0}(s)$ (5 – 77)

$\overline{P_{11}}(s) = A_{11}(s)\overline{P_0}(s)$ (5 – 78)

$$\overline{P_{12}}(s) = A_{12}(s)\overline{P_0}(s) \quad (5-79)$$

$$\overline{P_{13}}(s) = A_{13}(s)\overline{P_0}(s), \quad (5-80)$$

where:

$A_1(s) = \beta_1[N_1^{-1}(s) - \beta_1\bar{F}(s) - \beta_2 N_1^{-1}(s)\bar{F}(s) - \beta_4(s)\bar{U}(s)]^{-1}$
$A_2(s) = \beta_2[N_2^{-1}(s) - \beta_1\bar{F}(s) - \beta_3 N_1^{-1}(s)\bar{H}(s) - \beta_4(s)\bar{U}(s)]^{-1}$
$A_3(s) = \beta_3[N_3^{-1}(s) - \beta_2\bar{G}(s) - \beta_2 N_3^{-1}(s)\bar{F}(s) - \beta_4(s)\bar{U}(s)]^{-1}$
$A_4(s) = \beta_1 N_4(s)A_2(s), A_5(s) = \beta_1 N_5(s)A_1(s), A_6(s) = \beta_2 N_6(s)A_1(s)$
$A_7(s) = \beta_4 N_7(s)A_3(s), A_8(s) = \beta_3 N_8(s)A_2(s)$
$A_9(s) = \beta_4 N_9(s)A_2(s), A_{10}(s) = \beta_4 N_{10}(s)A_1(s)$
$A_{11}(s) = \beta_2 N_{11}(s)A_3(s), A_{12}(s) = \beta_3 N_{12}(s)A_3(s), A_{13}(s) = \lambda N_{13}(s)$

6- Evaluation of Laplace transform of up and down state availability

The Laplace transform of the probability that the system is in up (operable) and down (failed) state at time 't' can be evaluated as follows:

$$\bar{P}_{up}(s) = \overline{P_0}(s) + \overline{P_1}(s) + \overline{P_2}(s) + \overline{P_3}(s) + \overline{P_{13}}(s)$$

$$= \frac{1}{A(s)}[1 + A_1(s) + A_2(s) + A_3(s) + A_{13}(s)] \quad (6-1)$$

$$\bar{P}_{down}(s) = 1 - \bar{P}_{up}(s) \quad (6-2)$$

Steady state behavior of the system

Using Abel's Lemma in Laplace transform, viz $\lim_{s\to 0}[s.\bar{F}(s)] = \lim_{t\to\infty} F(t) = F\ (say)$, we get:

$$P_0 = \lim_{s\to 0} s.\overline{P_0}(s) = [1 + N_1^{-2}(0)N_1'(0)A_1(0)\bar{F}(\alpha_1+\alpha_2) - N_1^{-1}(0)A'_1(0)\bar{F}(\alpha_1+\alpha_2) - N_1^{-1}(0)A_1(0)\bar{F}'(s+\alpha_1+\alpha_2) + N_2^{-2}(0)N_2'(0)A_2(0)\bar{G}(\alpha_1+\alpha_2) - N_2^{-1}(0)A'_2(0)\bar{G}(\alpha_1+\alpha_2) - N_2^{-1}(0)A_2(0)\bar{G}'(\alpha_1+\alpha_2) - \lambda\bar{V}'(0)]^{-1}$$

$$P_1 = A_1(0)P_0, P_2 = A_2(0)P_0,\ P_3 = A_3(0)P_0,\ P_4 = A_4(0)P_0, P_5 = A_5(0)P_0,\ P_6 = A_6(0)P_0, P_7 = A_7(0)P_0 \quad (6-3)$$

Steady state availability of the system

$$P_{up} = \lim_{s\to 0} s.\bar{P}_{up}(s) = \lim_{s\to 0} s.[\bar{P}_0(s) + \bar{P}_1(s) + \bar{P}_2(s) + \bar{P}_3(s) + \bar{P}_{13}(s)] = [1 + A_1(0) + A_2(0) + A_3(0) + A_{13}(0)]P_{0,} \quad (6-4)$$

where:

$$A_1(0) = \alpha_1[N_1^{-1}(0) - \alpha_1\bar{F}(0) - \alpha_2 N_1^{-1}(0)N_2(0)\bar{G}(0)]^{-1}$$

$$A_1'(0) = \frac{\alpha_1[N_1^{-2}(0)N_1'(0) + \alpha_1\bar{F}'(0) + \alpha_2 N_1^{-2}(0)N_1'(0)N_2(0)\bar{G}(0) + \alpha_2 N_1^{-1}(0)N'_2(0)\bar{G}(0) + \alpha_2 N_1^{-1}(0)N_2(0)\bar{G}'(0)]}{[N_1^{-1}(0) - \alpha_1\bar{F}(0) - \alpha_2 N_1^{-1}(0)N_2(0)\bar{G}(0)]^2}$$

$$A_2(0) = \alpha_2[N_2^{-1}(0) - \alpha_1 N_1(0)N_2^{-1}(0)\bar{F}(0) - \alpha_2\bar{G}(0)]^{-1}$$

$$A_2'(0) = \frac{\alpha_2[N_2^{-2}(0)N_2'(0) + \alpha_1 N'_1(0)N_2^{-1}(0)\bar{F}(0) + \alpha_1 N_1(0)N_2^{-2}(0)N'_2(0)\bar{F}(0) + \alpha_1 N_1(0)N_2^{-1}(0)\bar{F}'(0) + \alpha_2\bar{G}'(0)]}{[N_2^{-1}(0) - \alpha_1 N_1(0)N_2^{-1}(0)\bar{F}(0) - \alpha_2\bar{G}(0)]^2}$$

$$A_3(0) = \alpha_1[N_1^{-1}(0) - \alpha_1\bar{F}(0) - \alpha_2 N_1^{-1}(0)N_2(0)\bar{G}(0)]^{-1}$$

$A_3'(0) =$

$$N_1(0) = (\alpha_1+\alpha_2)^{-1}[1 - \bar{F}(\alpha_1+\alpha_2)]$$

$$N_1'(0) = -(\alpha_1+\alpha_2)^{-2}[1 - \bar{F}(\alpha_1+\alpha_2)] - (\alpha_1+\alpha_2)^{-1}\bar{F}'(\alpha_1+\alpha_2)$$

$$N_2(0) = (\alpha_1+\alpha_2)^{-1}[1 - \bar{G}(\alpha_1+\alpha_2)]$$

$$N_2'(0) = -(\alpha_1+\alpha_2)^{-2}[1 - \bar{G}(\alpha_1+\alpha_2)] - (\alpha_1+\alpha_2)^{-1}\bar{G}'(\alpha_1+\alpha_2)$$

$$\overline{P_0}(s) = [s + \alpha_1 + \alpha_2 + \lambda - N_1^{-1}(s)A_1(s)\bar{F}(s+\alpha_1+\alpha_2) - N_2^{-1}(s)A_2(s)\bar{G}(s+\alpha_1+\alpha_2) - \lambda\bar{V}(s)]^{-1}$$

Mean Time To System Failure

$$MTSF = \lim_{s\to 0}\bar{P}_{up}(s) = [1 + A_1(0) + A_2(0) + A_7(0)]/[\alpha_1 + \alpha_2 + \lambda - N_1^{-1}(0)A_1(0)\bar{F}(\alpha_1+\alpha_2) - N_2^{-1}(0)A_2(0)\bar{G}(\alpha_1+\alpha_2) - \lambda\bar{V}(0)] \quad (6-5)$$

7- Particular Case

In this section the Laplace transform of up and down state availability, the mean time to system failure**(**MTSF), the steady state availability of the system and the expected total profit incurred to the system in the steady-state have been evaluated using supplementary variable technique with and without maintenance, when repair times follow exponential distribution.

Setting: $\bar{F}(s) = \frac{\beta_1}{s+\beta_1}, \bar{G}(s) = \frac{\beta_2}{s+\beta_2}, \bar{H}(s) = \frac{\beta_3}{s+\beta_3}, \bar{U}(s) = \frac{\beta_4}{s+\beta_4}$, and $\bar{V}(s) = \frac{\delta}{s+\delta}$in Equations (5-37) to (5-44) one gets:

$$\bar{P}_0(s) = (s+\delta)[(s+\beta_1+\beta_2+\beta_4+\alpha_3)(s+\alpha_1)(s+\alpha_2)(s+\alpha_4) - \alpha_1\beta_1(s+\alpha_2)(s+\alpha_4) - \alpha_2\beta_2(s+\alpha_1)(s+\alpha_4) - \alpha_4\beta_4(s+\alpha_1)(s+\alpha_2))((s+\beta_1+\beta_3+\beta_4+\alpha_2)(s+\alpha_1)(s+\alpha_3)(s+\alpha_4) - \alpha_1\beta_1(s+\alpha_3)(s+\alpha_4) - \alpha_3\beta_3(s+\alpha_1)(s+\alpha_4) - \alpha_4\beta_4(s+\alpha_1)(s+\alpha_3))((s+\beta_2+\beta_3+\beta_4+\alpha_3)(s+\alpha_2)(s+\alpha_3)(s+\alpha_4) - \alpha_2\beta_2(s+\alpha_3)(s+\alpha_4) - \alpha_3\beta_3(s+\alpha_2)(s+\alpha_4) - \alpha_4\beta_4(s+\alpha_2)(s+\alpha_3)]/D(s)$$
(7 – 1)

$$\bar{P}_1(s) = \beta_3(s+\delta)(s+\alpha_1)(s+\alpha_2)(s+\alpha_4)((s+\beta_1+\beta_3+\beta_4+\alpha_2)(s+\alpha_1)(s+\alpha_3)(s+\alpha_4) - \alpha_1\beta_1(s+\alpha_3)(s+\alpha_4) - \alpha_3\beta_3(s+\alpha_1)(s+\alpha_4) - \alpha_4\beta_4(s+\alpha_1)(s+\alpha_3))((s+\beta_2+\beta_3+\beta_4+\alpha_3)(s+\alpha_2)(s+\alpha_3)(s+\alpha_4) - \alpha_2\beta_2(s+\alpha_3)(s+\alpha_4) - \alpha_3\beta_3(s+\alpha_2)(s+\alpha_4) - \alpha_4\beta_4(s+\alpha_2)(s+\alpha_3)/D(s)$$
(7 – 2)

$$\bar{P}_2(s) = \beta_2(s+\delta)(s+\alpha_1)(s+\alpha_3)(s+\alpha_4)((s+\beta_1+\beta_2+\beta_4+\alpha_3)(s+\alpha_1)(s+\alpha_2)(s+\alpha_4) - \alpha_1\beta_1(s+\alpha_2)(s+\alpha_4) - \alpha_2\beta_2(s+\alpha_1)(s+\alpha_4) - \alpha_4\beta_4(s+\alpha_1)(s+\alpha_2))((s+\beta_2+\beta_3+\beta_4+\alpha_3)(s+\alpha_2)(s+\alpha_3)(s+\alpha_4) - \alpha_2\beta_2(s+\alpha_3)(s+\alpha_4) - \alpha_3\beta_3(s+\alpha_2)(s+\alpha_4) - \alpha_4\beta_4(s+\alpha_2)(s+\alpha_3))/D(s)$$
(7 – 3)

$$\bar{P}_3(s) = \beta_1(s+\delta)(s+\alpha_2)(s+\alpha_3)(s+\alpha_4)((s+\beta_1+\beta_2+\beta_4+\alpha_3)(s+\alpha_1)(s+\alpha_2)(s+\alpha_4) - \alpha_1\beta_1(s+\alpha_2)(s+\alpha_4) - \alpha_2\beta_2(s+\alpha_1)(s+\alpha_4) - \alpha_4\beta_4(s+\alpha_1)(s+\alpha_2))((s+\beta_1+\beta_3+\beta_4+\alpha_2)(s+\alpha_1)(s+\alpha_3)(s+\alpha_4) - \alpha_1\beta_1(s+\alpha_3)(s+\alpha_4) - \alpha_3\beta_3(s+\alpha_1)(s+\alpha_4) - \alpha_4\beta_4(s+\alpha_1)(s+\alpha_3))/D(s)$$
(7 - 4)

$$\bar{P}_4(s) = \beta_1\beta_2(s+\delta)(s+\alpha_3)(s+\alpha_4)((s+\beta_1+\beta_2+\beta_4+\alpha_3)(s+\alpha_1)(s+\alpha_2)(s+\alpha_4) - \alpha_1\beta_1(s+\alpha_2)(s+\alpha_4) - \alpha_2\beta_2(s+\alpha_1)(s+\alpha_4) - \alpha_4\beta_4(s+\alpha_1)(s+\alpha_2))((s+\beta_2+\beta_3+\beta_4+\alpha_3)(s+\alpha_2)(s+\alpha_3)(s+\alpha_4) - \alpha_2\beta_2(s+\alpha_3)(s+\alpha_4) - \alpha_3\beta_3(s+\alpha_2)(s+\alpha_4) - \alpha_4\beta_4(s+\alpha_2)(s+\alpha_3))/D(s) \quad (7-5)$$

$$\bar{P}_5(s) = \beta_1\beta_3(s+\delta)(s+\alpha_2)(s+\alpha_4)((s+\beta_1+\beta_3+\beta_4+\alpha_2)(s+\alpha_1)(s+\alpha_3)(s+\alpha_4) - \alpha_1\beta_1(s+\alpha_3)(s+\alpha_4) - \alpha_3\beta_3(s+\alpha_1)(s+\alpha_4) - \alpha_4\beta_4(s+\alpha_1)(s+\alpha_3))((s+\beta_2+\beta_3+\beta_4+\alpha_3)(s+\alpha_2)(s+$$

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$\alpha_3)(s+\alpha_4)-\alpha_2\beta_2(s+\alpha_3)(s+\alpha_4)-\alpha_3\beta_3(s+\alpha_2)(s+\alpha_4)-\alpha_4\beta_4(s+\alpha_2)(s+\alpha_3))/D(s)$ (7 - 6)

$\overline{P}_6(s)=\beta_2\beta_3(s+\delta)(s+\alpha_1)(s+\alpha_4)((s+\beta_1+\beta_3+\beta_4+\alpha_2)(s+\alpha_1)(s+\alpha_3)(s+\alpha_4)-\alpha_1\beta_1(s+\alpha_3)(s+\alpha_4)-\alpha_3\beta_3(s+\alpha_1)(s+\alpha_4)-\alpha_4\beta_4(s+\alpha_1)(s+\alpha_3))((s+\beta_2+\beta_3+\beta_4+\alpha_3)(s+\alpha_2)(s+\alpha_3)(s+\alpha_4)-\alpha_2\beta_2(s+\alpha_3)(s+\alpha_4)-\alpha_3\beta_3(s+\alpha_2)(s+\alpha_4)-\alpha_4\beta_4(s+\alpha_2)(s+\alpha_3))/D(s)$ (7 - 7)

$\overline{P}_7(s)=\beta_4\beta_1(s+\delta)(s+\alpha_2)(s+\alpha_3)((s+\beta_1+\beta_2+\beta_4+\alpha_3)(s+\alpha_1)(s+\alpha_2)(s+\alpha_4)-\alpha_1\beta_1(s+\alpha_2)(s+\alpha_4)-\alpha_2\beta_2(s+\alpha_1)(s+\alpha_4)-\alpha_4\beta_4(s+\alpha_1)(s+\alpha_2))((s+\beta_1+\beta_3+\beta_4+\alpha_2)(s+\alpha_1)(s+\alpha_3)(s+\alpha_4)-\alpha_1\beta_1(s+\alpha_3)(s+\alpha_4)-\alpha_3\beta_3(s+\alpha_1)(s+\alpha_4)-\alpha_4\beta_4(s+\alpha_1)(s+\alpha_3))/D(s)$ (7 - 8)

$\overline{P}_8(s)=\beta_3\beta_2(s+\delta)(s+\alpha_1)(s+\alpha_4)((s+\beta_1+\beta_2+\beta_4+\alpha_3)(s+\alpha_1)(s+\alpha_2)(s+\alpha_4)-\alpha_1\beta_1(s+\alpha_2)(s+\alpha_4)-\alpha_2\beta_2(s+\alpha_1)(s+\alpha_4)-\alpha_4\beta_4(s+\alpha_1)(s+\alpha_2))((s+\beta_2+\beta_3+\beta_4+\alpha_3)(s+\alpha_2)(s+\alpha_3)(s+\alpha_4)-\alpha_2\beta_2(s+\alpha_3)(s+\alpha_4)-\alpha_3\beta_3(s+\alpha_2)(s+\alpha_4)-\alpha_4\beta_4(s+\alpha_2)(s+\alpha_3))/D(s)$ (7 - 9)

$\overline{P}_9(s)=\beta_{43}\beta_2(s+\delta)(s+\alpha_1)(s+\alpha_3)((s+\beta_1+\beta_2+\beta_4+\alpha_3)(s+\alpha_1)(s+\alpha_2)(s+\alpha_4)-\alpha_1\beta_1(s+\alpha_2)(s+\alpha_4)-\alpha_2\beta_2(s+\alpha_1)(s+\alpha_4)-\alpha_4\beta_4(s+\alpha_1)(s+\alpha_2))((s+\beta_2+\beta_3+\beta_4+\alpha_3)(s+\alpha_2)(s+\alpha_3)(s+\alpha_4)-\alpha_2\beta_2(s+\alpha_3)(s+\alpha_4)-\alpha_3\beta_3(s+\alpha_2)(s+\alpha_4)-\alpha_4\beta_4(s+\alpha_2)(s+\alpha_3))/D(s)$ (7 - 10)

$\overline{P}_{10}(s)=\beta_4\beta_3(s+\delta)(s+\alpha_1)(s+\alpha_2)((s+\beta_1+\beta_3+\beta_4+\alpha_2)(s+\alpha_1)(s+\alpha_3)(s+\alpha_4)-\alpha_1\beta_1(s+\alpha_3)(s+\alpha_4)-\alpha_3\beta_3(s+\alpha_1)(s+\alpha_4)-\alpha_4\beta_4(s+\alpha_1)(s+\alpha_3))((s+\beta_2+\beta_3+\beta_4+\alpha_3)(s+\alpha_2)(s+\alpha_3)(s+\alpha_4)-\alpha_2\beta_2(s+\alpha_3)(s+\alpha_4)-\alpha_3\beta_3(s+\alpha_2)(s+\alpha_4)-\alpha_4\beta_4(s+\alpha_2)(s+\alpha_3))/D(s)$ (7 - 11)

$\overline{P}_{11}(s)=\beta_2\beta_1(s+\delta)(s+\alpha_3)(s+\alpha_4)((s+\beta_1+\beta_2+\beta_4+\alpha_3)(s+\alpha_1)(s+\alpha_2)(s+\alpha_4)-\alpha_1\beta_1(s+\alpha_2)(s+\alpha_4)-\alpha_2\beta_2(s+\alpha_1)(s+\alpha_4)-\alpha_4\beta_4(s+\alpha_1)(s+\alpha_2))((s+\beta_1+\beta_3+\beta_4+\alpha_2)(s+\alpha_1)(s+\alpha_3)(s+\alpha_4)-\alpha_1\beta_1(s+\alpha_3)(s+\alpha_4)-\alpha_3\beta_3(s+\alpha_1)(s+\alpha_4)-\alpha_4\beta_4(s+\alpha_1)(s+\alpha_3))/D(s)$ (7 - 12)

$\overline{P}_{12}(s)=\beta_3\beta_1(s+\delta)(s+\alpha_2)(s+\alpha_4)((s+\beta_1+\beta_2+\beta_4+\alpha_3)(s+\alpha_1)(s+\alpha_2)(s+\alpha_4)-\alpha_1\beta_1(s+\alpha_2)(s+\alpha_4)-\alpha_2\beta_2(s+\alpha_1)(s+\alpha_4)-\alpha_4\beta_4(s+\alpha_1)(s+\alpha_2))((s+\beta_1+\beta_3+\beta_4+\alpha_2)(s+\alpha_1)(s+\alpha_3)(s+\alpha_4)-\alpha_1\beta_1(s+\alpha_3)(s+\alpha_4)-\alpha_3\beta_3(s+\alpha_1)(s+\alpha_4)-\alpha_4\beta_4(s+\alpha_1)(s+\alpha_3))/D(s)$ (7 - 13)

$\overline{P}_{13}(s)=\lambda((s+\beta_1+\beta_2+\beta_4+\alpha_3)(s+\alpha_1)(s+\alpha_2)(s+\alpha_4)-\alpha_1\beta_1(s+\alpha_2)(s+\alpha_4)-\alpha_2\beta_2(s+\alpha_1)(s+\alpha_4)-\alpha_4\beta_4(s+\alpha_1)(s+\alpha_2))((s+\beta_1+\beta_3+\beta_4+\alpha_2)(s+\alpha_1)(s+\alpha_3)(s+\alpha_4)-\alpha_1\beta_1(s+\alpha_3)(s+\alpha_4)-\alpha_3\beta_3(s+\alpha_1)(s+\alpha_4)-\alpha_4\beta_4(s+\alpha_1)(s+\alpha_3))((s+\beta_2+\beta_3+\beta_4+\alpha_3)(s+\alpha_2)(s+\alpha_3)(s+\alpha_4)-\alpha_2\beta_2(s+\alpha_3)(s+\alpha_4)-\alpha_3\beta_3(s+\alpha_2)(s+\alpha_4)-\alpha_4\beta_4(s+\alpha_2)(s+\alpha_3))/D(s)$
(7 - 14)

Therefore, the Laplace transform of the probability that the system is in up (operable) and down (failed) state at time 't' can be evaluated as follows:

$$\bar{P}_{up}(s)=\bar{P}_0(s)+\bar{P}_1(s)+\bar{P}_2(s)+\bar{P}_3(s)+\bar{P}_{13}(s)$$

$$\bar{P}_{up}(s)=N(s)/D(s) \qquad (7-15)$$

$$\bar{P}_{down}(s)=1-\bar{P}_{up}(s) \qquad (7-16)$$

Busy period analysis

The initial conditions for this problem are the same as for the reliability case. Then the probability that the system is in statesS_1-S_{13} is given by:
$\overline{P}_B(s)=\frac{1}{s}-\left(\overline{P}_0(s)+\overline{P}_{13}(s)\right)=\frac{1}{s}-\quad(s+\delta+\lambda)((s+\beta_1+\beta_2+\beta_4+\alpha_3)(s+\alpha_1)(s+\alpha_2)(s+\alpha_4)-\alpha_1\beta_1(s+\alpha_2)(s+\alpha_4)-\alpha_2\beta_2(s+\alpha_1)(s+\alpha_4)-\alpha_4\beta_4(s+\alpha_1)(s+\alpha_2))\quad((s+\beta_1+\beta_3+\beta_4+\alpha_2)(s+\alpha_1)(s+\alpha_3)(s+\alpha_4)-\alpha_1\beta_1(s+\alpha_3)(s+\alpha_4)-\alpha_3\beta_3(s+\alpha_1)(s+\alpha_4)-\alpha_4\beta_4(s+\alpha_1)(s+\alpha_3))\quad((s+\beta_2+\beta_3+\beta_4+\alpha_3)(s+\alpha_2)(s+\alpha_3)(s+\alpha_4)-\alpha_2\beta_2(s+\alpha_3)(s+\alpha_4)-\alpha_3\beta_3(s+\alpha_2)(s+\alpha_4)-\alpha_4\beta_4(s+\alpha_2)(s+\alpha_3))/D(s)$ (7 – 17)

The expected frequency of preventive maintenance:

The initial conditions for this problem are the same as for the reliability case. Then the expected frequency of preventive maintenance is given by
$\overline{K}(s)=\overline{P}_{13}(s)=\quad\lambda((s+\beta_1+\beta_2+\beta_4+\alpha_3)(s+\alpha_1)(s+\alpha_2)(s+\alpha_4)-\alpha_1\beta_1(s+\alpha_2)(s+\alpha_4)-\alpha_2\beta_2(s+\alpha_1)(s+\alpha_4)-\alpha_4\beta_4(s+\alpha_1)(s+\alpha_2))((s+\beta_1+\beta_3+\beta_4+\alpha_2)(s+\alpha_1)(s+\alpha_3)(s+\alpha_4)-\alpha_1\beta_1(s+\alpha_3)(s+\alpha_4)-\alpha_3\beta_3(s+\alpha_1)(s+\alpha_4)-\alpha_4\beta_4(s+\alpha_1)(s+\alpha_3))((s+\beta_2+\beta_3+\beta_4+\alpha_3)(s+\alpha_2)(s+\alpha_3)(s+\alpha_4)-\alpha_2\beta_2(s+\alpha_3)(s+\alpha_4)-\alpha_3\beta_3(s+\alpha_2)(s+\alpha_4)-\alpha_4\beta_4(s+\alpha_2)(s+\alpha_3))/D(s)$
(7 - 18)

Mean Time to System Failure with maintenance

$$MTSF_{With}=\lim_{s\to 0}\bar{P}_{up}(s)=N/D \qquad (7-19)$$

Mean Time to System Failure without maintenance
$$MTSF_{Wout}=\lim_{s\to 0}\bar{P}_{up}(s)=N_{Wout}/D_{Wout} \qquad (7-20)$$

The steady state availability of the system with maintenance is given by:
$$A_{With}(\infty)=A(\infty)=N/DD \qquad (7-21)$$

The steady state availability of the system without maintenance is given by:
$$A_{Wout}(\infty)=N_{Wout}/DD_{Wout}] \qquad (7-22)$$

The steady state busy period of the system with maintenance is given by:
$$B(\infty)=1-(P_0+P_7)=1-NB/DD \qquad (7-23)$$
Where
$$NB=\quad(\lambda+\delta)(\alpha_1\alpha_2\alpha_4(\beta_1+\alpha_3+\beta_2+\beta_4)-\alpha_1\alpha_2\beta_2\alpha_4-\alpha_1\alpha_2\alpha_4\beta_4-\alpha_1\alpha_2\beta_1\alpha_4)\,(\alpha_1\alpha_3\alpha_4(\alpha_2+\beta_1+\beta_3+\beta_4)-\alpha_1\alpha_3\alpha_4\beta_3-\alpha_1\alpha_3\alpha_4\beta_4-\alpha_1\beta_1\alpha_3\alpha_4)\,(\alpha_2\alpha_3\alpha_4(\alpha_1+\beta_2+\beta_3+\beta_4)-\alpha_2\alpha_3\alpha_4\beta_3-\alpha_2\alpha_3\alpha_4\beta_4-\alpha_2\alpha_3\beta_2\alpha_4)$$

The steady state busy period of the system without maintenance is given by:
$$B_{Wout}(\infty)=1-(P_0+P_7)=1-\langle(\alpha_1\alpha_2\alpha_4(\beta_1+\alpha_3+\beta_2+\beta_4)-\alpha_1\alpha_2\beta_2\alpha_4-\alpha_1\alpha_2\alpha_4\beta_4-\alpha_1\alpha_2\beta_1\alpha_4)\,(\alpha_1\alpha_3\alpha_4(\alpha_2+\beta_1+\beta_3+\beta_4)-\alpha_1\alpha_3\alpha_4\beta_3-\alpha_1\alpha_3\alpha_4\beta_4-\alpha_1\beta_1\alpha_3\alpha_4)\,(\alpha_2\alpha_3\alpha_4(\alpha_1+\beta_2+\beta_3+\beta_4)-\alpha_2\alpha_3\alpha_4\beta_3-\alpha_2\alpha_3\alpha_4\beta_4-\alpha_2\alpha_3\beta_2\alpha_4)\;/DD_{Wout}\rangle \qquad (7-24)$$

The steady state expected frequency of the system is given by:
$$K(\infty)=P_7=NK/DD \qquad (7-25)$$
Where
$$NK=\quad\lambda(\alpha_1\alpha_2\alpha_4(\beta_1+\alpha_3+\beta_2+\beta_4)-\alpha_1\alpha_2\beta_2\alpha_4-\alpha_1\alpha_2\alpha_4\beta_4-\alpha_1\alpha_2\beta_1\alpha_4)\,(\alpha_1\alpha_3\alpha_4(\alpha_2+\beta_1+\beta_3+\beta_4)-\alpha_1\alpha_3\alpha_4\beta_3-\alpha_1\alpha_3\alpha_4\beta_4-\alpha_1\beta_1\alpha_3\alpha_4)\,(\alpha_2\alpha_3\alpha_4(\alpha_3+\beta_2+\beta_3+\beta_4)-\alpha_2\alpha_3\alpha_4\beta_3-\alpha_2\alpha_3\alpha_4\beta_4-\alpha_2\alpha_3\beta_2\alpha_4)$$

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The expected total profit incurred to the system in the steady-state is given by:

$$PF_{With}(\infty) = R.A(\infty) - C_1 B(\infty) - C_2 K(\infty) \quad (7-26)$$

The expected total profit without maintenance is given by:

$$PF_{Without}(\infty) = RA(\infty) - C_1 B(\infty) \quad (7-27)$$

8- Numerical computation

a- Availability analysis

If we put: $\beta_1 = .03, \beta_2 = .04, \beta_3 = .05, \beta_4 = .06, \alpha_1 = .3, \alpha_2 = .4, \alpha_3 = .5, \alpha_4 = .6, \lambda = .02, \delta = .2$ (8 – 1)

In Equation (7 - 9), we get:

$$\bar{P}_{up}(s) = (((s+.2+.02)((s+.03+.04+.06+.5)(s+.3)(s+.4)(s+.6) - .3.03(s+.4)(s+.6) - .4.04(s+.3)(s+.6) - .6.06(s+.3)(s+.4))((s+.03+.05+.06+.4)(s+.3)(s+.5)(s+.6) - .3.03(s+.5)(s+.6) - .5.05(s+.3)(s+.6) - .6.06(s+.3)(s+.5))((s+.04+.05+.06+.5)(s+.4)(s+.5)(s+.6) - .4.04(s+.5)(s+.6) - .5.05(s+.4)(s+.6) - .6.06(s+.4)(s+.5)) + .05(s+.2)(s+.3)(s+.4)(s+.6)((s+.03+.05+.06+.4)(s+.3)(s+.5)(s+.6) - .3.03(s+.5)(s+.6) - .5.05(s+.3)(s+.6) - .6.06(s+.3)(s+.5))((s+.04+.05+.06+.5)(s+.4)(s+.5)(s+.6) - .4.04(s+.5)(s+.6) - .5.05(s+.4)(s+.6) - .6.06(s+.4)(s+.5)) + .04(s+.2)(s+.3)(s+.5)(s+.6)((s+.03+.04+.06+.5)(s+.3)(s+.4)(s+.6) - .3.03(s+.4)(s+.6) - .4.04(s+.3)(s+.6) - .6.06(s+.3)(s+.4))((s+.04+.05+.06+.5)(s+.4)(s+.5)(s+.6) - .4.04(s+.5)(s+.6) - .5.05(s+.4)(s+.6) - .6.06(s+.4)(s+.5)) + .03(s+.2)(s+.4)(s+.5)(s+.6)((s+.03+.04+.06+.5)(s+.3)(s+.4)(s+.6) - .3.03(s+.4)(s+.6) - .4.04(s+.3)(s+.6) - .6.06(s+.3)(s+.4))((s+.03+.05+.06+.4)(s+.3)(s+.5)(s+.6) - .3.03(s+.5)(s+.6) - .5.05(s+.3)(s+.6) - .6.06(s+.3)(s+.5)))/((s+0.8952)(s+0.85676)(s+0.81554)(s+0.55106)(s+0.54422)(s+0.50859)(s+0.44023)(s+0.38518)(s+0.35363)(s+0.30783)(s+0.26893)(s+0.23692)(s+0.20791)(s+4.8000\times 10^{-2})))/(s^{14}+6.42\,s^{13}+18.711s^{12}+32.788s^{11}+38.566s^{10}+32.177s^{9}+19.613\,s^{8}+8.8588s^{7}+2.9726\,s^{6}+0.73513s^{5}+0.13127\,s^{4}1.6294\times 10^{-2}s^{3}+1.3129\times 10^{-3}\,s^{2}+6.008\times 10^{-5}s++1.1197\times 10^{-6}) \quad (8-2)$$

By taking inverse Laplace transform of Equation (8 – 2) one may get the probability that the system is in up- state at time 't'.

$$P_{up}(t) = 6.2317\times 10^{-3}exp(-0.50673t) + 0.02612exp(-0.26693t) + 0.95565exp(-8.4485\times 10^{-3}t) + 1.5494\times 10^{-4}exp(-0.54395t) - 1.1185\times 10^{-3}exp(-0.55074t) + 2.7527\times 10^{-2}exp(-0.30289t) + 2.9847\times 10^{-3}exp(-0.35335t) + 2.7791\times 10^{-2}exp(-0.37958t) - 3.349\times 10^{-3}exp(-0.81533t) - 6.3750\times 10^{-3}exp(-0.20722t) - 4.7522\times 10^{-3}exp(-0.85556t) - 3.567\times 10^{-4}exp(-0.43988t) - 3.3681\times 10^{-2}exp(-0.89309t) + 3.1736\times 10^{-3}exp(-0.23631t)$$

(8 – 3)

Setting $t = 0, 1, 2, \dots\dots\dots$, in Equation (8 – 3), one can compute Table 1. Variation of availability w.r.t. time is shown in Figure 2.

Table (1): Variation of the availability with respect to time

Time (t)	Availability with maintenance	Availability without maintenance
0	1.0	1.0
1	0.99206	0.94959
2	0.97685	0.91339
3	0.96074	0.88631
4	0.94576	0.86524
5	0.93232	0.84825
6	0.92032	0.83406
7	0.90948	0.82185
8	0.89955	0.81105
9	0.89031	0.80126
10	0.8816	0.79222

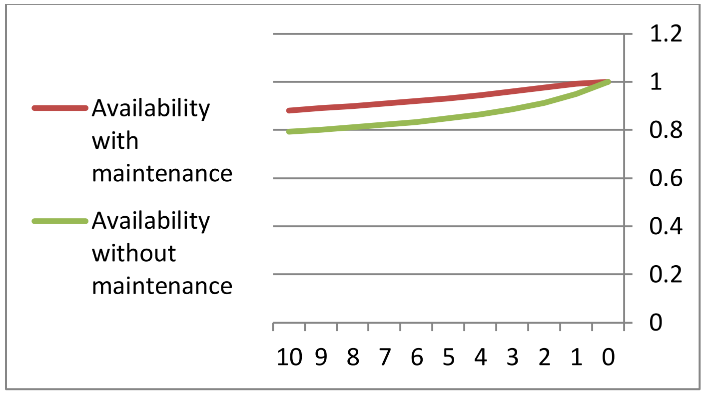

Figure (2): Variation of the availability with respect to time

b- Cost analysis

The expected total gain of the system incurred to the system in the interval (0 t] is given by:

ETG= $G(t)$**= total revenue - total cost**

$$G(t) = R\mu_{up}(t) - C_1\mu_B(t) - C_2\mu_k(t) \quad (8-4)$$

Where:

$G(t)$: the expected total gain in interval (0, t]

R : the revenue per unit of time of the system,

$\mu_{up}(t)$: the mean up time in interval (0, t]

$\mu_B(t)$: the expected busy period for repair of the system

$\mu_k(t)$: the expected busy period for preventive maintenance

C_1, C_2, : the service costs per unit of time for repair from failed states and for preventive maintenance

The mean up time is given by:

From Equation (8 – 3) , one may get:

$$\mu_{up}(t) = \int_0^t P_{up}(t)dt = 2.0309\times 10^{-3}\exp(-0.55074t) - 9.7853\times 10^{-2}\exp(-0.26693t) - 113.11\exp(-8.4485\times 10^{-3}t) - 2.8484\times 10^{-4}\exp(-0.54395t) - 1.2298\times 10^{-2}\exp(-0.50673t) - 9.0881\times 10^{-2}\exp(-0.30289t) - 8.4469\times 10^{-3}\exp(-0.35335t) - 7.3215\times 10^{-2}\exp(-0.37958t) + 4.1075\times 10^{-3}\exp(-0.81533t) + 3.0764\times 10^{-2}\exp(-0.20722t) + 5.5545\times 10^{-3}\exp(-0.85556t) + 8.109\times 10^{-4}\exp(-0.43988t) - 3.7713\times 10^{-2}\exp(-0.89309t) - 1.3430\times 10^{-2}\exp(-0.23631t) + 113.41 \quad (8-5)$$

Taking the inverse of Laplace transform of Equation (7 - 11), one may get the probability that the system is in state $(S_1 - S_6)$ at time 't'

$$P_B(t) = 11 + 1.8996 \times 10^{-2}\exp(-0.20722t) - 2.3404 \times 10^{-2}\exp(-0.26693t) - 0.7699\exp(-8.4485 \times 10^{-3}t) - 3.5840 \times 10^{-3}\exp(-0.54395t) - 4.0523 \times 10^{-3}\exp(-0.55074t) - 5.8761 \times 10^{-2}\exp(-0.30289t) - 3.8878 \times 10^{-3}\exp(-0.35335t) - 6.8909 \times 10^{-2}\exp(-0.37958t) - 3.3408 \times 10^{-3}\exp(-0.81533t) - 2.5011 \times 10^{-2}\exp(-0.50673t) - 1.8726 \times 10^{-2}\exp(-0.85556t) - 4.4354 \times 10^{-3}\exp(-0.43988t) - 3.0524 \times 10^{-2}\exp(-0.89309t) - 4.4559 \times 10^{-3}\exp(-0.23631t)$$

And hence,

$$\mu_B(t) = t + 4.9358 \times 10^{-2}\exp(-0.50673t) + 8.7678 \times 10^{-2}\exp(-0.26693t) + 91.129\exp(-8.4485 \times 10^{-3}t) + 6.5888 \times 10^{-3}\exp(-0.54395t) + 7.3579 \times 10^{-3}\exp(-0.55074t) + 0.194\exp(-0.30289t) + 1.1003 \times 10^{-2}\exp(-0.35335t) + 0.18154\exp(-0.37958t) + 4.0975 \times 10^{-3}\exp(-0.81533t) - 9.1671 \times 10^{-2}\exp(-0.20722t) + 2.1887 \times 10^{-2}\exp(-0.85556t) + 1.0083 \times 10^{-2}\exp(-0.43988t) + 3.4178 \times 10^{-2}\exp(-0.89309t) + 1.8856 \times 10^{-2}\exp(-0.23631t) - 91.664$$
(8 – 6)

Taking the inverse of Laplace transform of Equation (7 – 12), one may get the probability that the system is in state S_7 at time 't'

$$P_K(t) = 7.2786 \times 10^{-2}exp(-8.4485 \times 10^{-3}t) - 9.9742 \times 10^{-3}exp(-0.26693t) - 1.7446 \times 10^{-3}exp(-0.50673t) - 2.2127 \times 10^{-4}exp(-0.54395t) - 2.4504 \times 10^{-4}exp(-0.55074t) - 1.4178 \times 10^{-2}exp(-0.30289t) - 5.8309 \times 10^{-4}exp(-0.35335t) - 8.6363 \times 10^{-3}exp(-0.37958t) - 1.1223 \times 10^{-4}exp(-0.81533t) - 2.9728 \times 10^{-2}exp(-0.20722t) - 5.8927 \times 10^{-4}exp(-0.85556t) - 4.0344 \times 10^{-4}exp(-0.43988t) - 9.0697 \times 10^{-4}exp(-0.89309t) - 5.464 \times 10^{-3}exp(-0.23631t)$$

and hence,

$$\mu_K(t) = 3.4429 \times 10^{-3}exp(-0.50673t) + 3.7366 \times 10^{-2}exp(-0.26693t) - 8.6153exp(-8.4485 \times 10^{-3}t) + 4.0678 \times 10^{-4}exp(-0.54395t) + 4.4493 \times 10^{-4}exp(-0.55074t) + 4.6809 \times 10^{-2}exp(-0.30289t) + 1.6502 \times 10^{-3}exp(-0.35335t) + 2.2752 \times 10^{-2}exp(-0.37958t) + 1.3765 \times 10^{-4}exp(-0.81533t) + 0.14346exp(-0.20722t) + 6.8875 \times 10^{-4}exp(-0.85556t) + 9.1716 \times 10^{-4}exp(-0.43988t) + 1.0155 \times 10^{-3}exp(-0.89309t) + 2.3122 \times 10^{-2}exp(-0.23631t) + 8.333$$
(8 – 7)

By substituting Equations (8 – 5)-(8 – 7) in Equation (8 – 4), one may get:

$$G(t) = R\mu_{up}(t) - C_1\mu_B(t) - C_2\mu_k(t) \qquad (8-8)$$

Setting values for the cost coefficient C_1, C_2 and revenue R one can get the expected total gain in the interval (0, t] for different values of 't' .Setting t =1, 2, 3,….. in Equation (8 – 8) for $R = 1000, C_1 = 100, C_2 = 50$, one compute Table (2). Figure (3) show expected total profit increase in the interval (0, t] for three cases.

Table (2):Expected total profit in the interval (0, t] for different values of 't'

t	PF(t) with maintenance	PF(t) without maintenance
0	9.1548	-0.141
1	1045.3	968.1
2	2034.6	1886.1
3	2992.1	2767.9
4	3925.9	3622.5
5	4841.2	4455.7
6	5741.2	5271.5
7	6627.9	6072.7
8	7502.9	6861.1
9	8367.2	7638.1
10	9221.7	8404.8

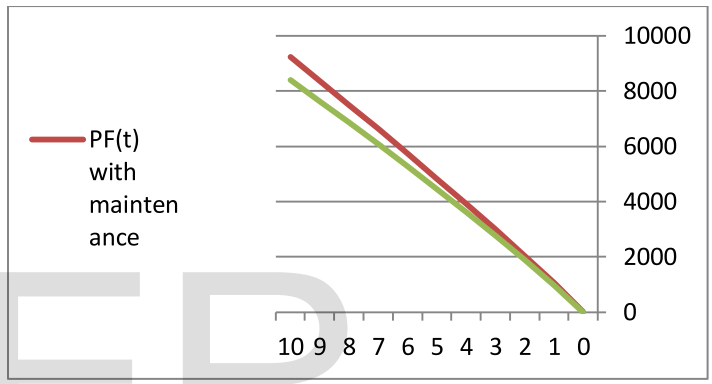

Figure (3):Expected total profit increase in the interval (0, t] for first case

C- MTSF Analysis

By using Equation(8 – 1) and setting $\alpha_1 = .01, .02, .03, \ldots\ldots\ldots$, in Equations (7 – 13), (7 – 14) one can compute Table 3. Variation of MTSF of the system with and without maintenance for different values of β_1 is shown in Figure 4.

Table (3): Variation of MTSF with different values of α_1

β_1	MTSF of the system with maintenance	MTSF of the system withoutmaintenance
.0	258.42	238.54
.01	247.13	228.41
.02	236.96	219.28
.03	227.75	211.00
.04	219.35	203.46
.05	211.68	196.56
.06	204.62	190.21
.07	198.12	184.36
.08	192.11	178.95
.09	186.53	173.92
.10	181.33	169.24

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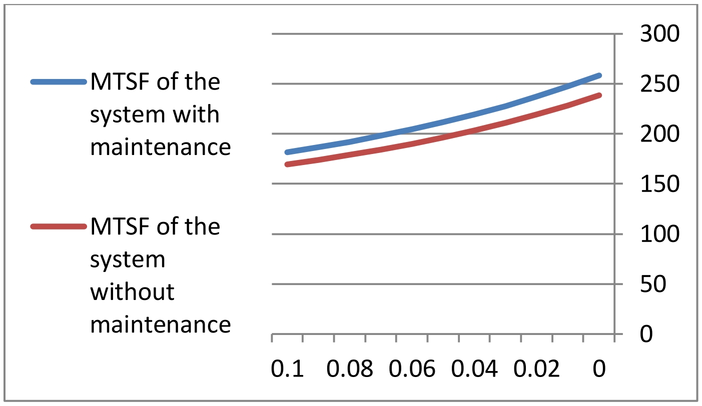

Figure (4): Variation of MTSF for different values of α_1

By using Equation(8 – 1) and setting $\alpha_1 = .01, .02, .03, \dots \dots \dots$, in Equations (7 – 13), (7 – 14) one can compute Table 3. Variation of MTSF of the system with and without maintenance for different values of α_1 is shown in Figure 4.

Table (3): Variation of MTSF with different values of α_1

α_1	MTSF of the system with maintenance	MTSF of the system without maintenance
.0	0.0	0.0
.01	159.37	147.65
.02	205.68	190.55
.03	227.74	210.99
.04	240.65	222.95
.05	249.12	230.80
.06	255.11	236.34
.07	259.56	240.47
.08	263.01	243.66
.09	265.75	246.20
.10	267.99	248.27

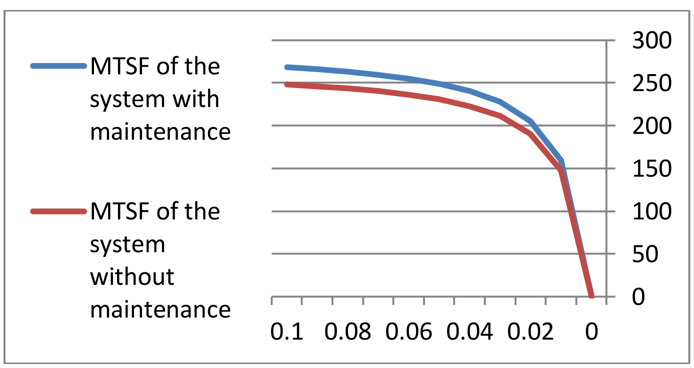

Figure (4): Variation of MTSF for different values of α_1

d- Steady-state availability

If we put: α_1=.01, .02,.03,……, at constant α_2=,04,β_1=.05,β_2=.06,λ=.02,δ=.02, in Equations (7 – 15),(7 – 16) we get Table4. Variation of steady state availability with respect to failure rate of type I α_1 is shown in Figure 5.

Table (4): Variation of steady-state availability with respect to failure rate

β_1	Availability of the system with maintenance	Availability of the system without maintenance
.0	0.97014	0.96773
.01	0.95922	0.95606
.02	0.94906	0.94526
.03	0.93958	0.93524
.04	0.93072	0.92592
.05	0.92242	0.91722
.06	0.91462	0.90909
.07	0.90729	0.90146
.08	0.90037	0.89431
.09	0.89384	0.88757
.1	0.88767	0.88122

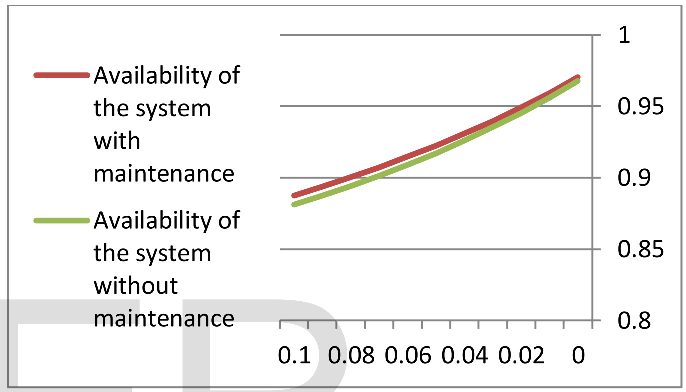

Figure (5): Variation of steady-state Busy period with and without maintenance for different values of α_1.

If we put: α_1=.01, .02,.03,……, at constant α_2=,04,β_1=.05,β_2=.06,λ=.02,δ=.02, in Equations (7 – 15),(7 – 16) we get Table4. Variation of steady state availability with respect to failure rate of type I α_1 is shown in Figure 5.

Table (4): Variation of steady-state availability with respect to repair rate

α_1	Steady-state availability of the system with maintenance	Steady-state availability of the system without maintenance
.0	0.0	0.0
.01	0.7363	0.73171
.02	0.77994	0.77364
.03	0.80986	0.80292
.04	0.83164	0.82452
.05	0.84821	0.84112
.06	0.86124	0.85427
.07	0.87176	0.86495
.08	0.88042	0.87379
.09	0.88768	0.88123
.1	0.89385	0.88757

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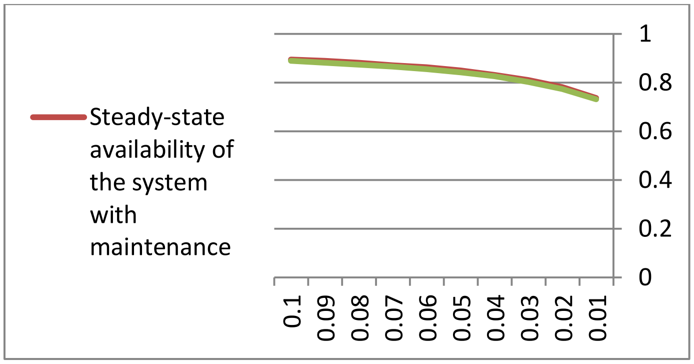

Figure (5): Variation of steady-state Busy period with and without maintenance for different values of α_1.

e- Steady-state Busy Period

If we put: α_1=.01, .02,.03,……, at constant α_2=,04,β_1=.05,β_2=.06, λ=.02, δ=.02 in Equations (7 – 17),(7 – 18) we get Table5. Variation of steady state availability with respect to failure rate β_1 is shown in Figure 6.

Table (5): Variation of steady-state busy period with respect to failure rate of type I

β_1	Steady-state busy period of the system with maintenance	Steady-state busy period of the system without maintenance
.0	0.17911	0.19355
.01	0.20864	0.22481
.02	0.23612	0.25374
.03	0.26175	0.28058
.04	0.28572	0.30556
.05	0.30818	0.32886
.06	0.32928	0.35065
.07	0.34912	0.37107
.08	0.36782	0.39024
.09	0.38548	0.40828
.1	0.40218	0.42529

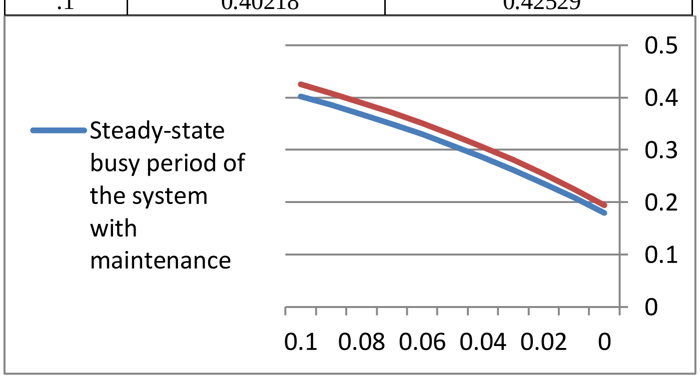

Figure (6):Variation of steady-state busy period with and without maintenance for different values of α_1

If we put: α_1=.01, .02,.03,……, at constant α_2=,04,β_1=.05,β_2=.06, λ=.02, δ=.02 in Equations (7 – 17),(7 – 18) we get Table5. Variation of steady state availability with respect to repair rate α_1 is shown in Figure 6.

Table (5): Variation of steady-state busy period with respect to failure rate of type I

α_1	Steady-state busy period of the system with maintenance	Steady-state busy period of the system without maintenance
..0	1.0	1.0
.01	0.81164	0.82578
.02	0.69359	0.71347
.03	0.61268	0.63504
.04	0.55375	0.57717
.05	0.50893	0.53271
.06	0.47368	0.49749
.07	0.44525	0.46889
.08	0.42181	0.44521
.09	0.40217	0.42529
.1	0.38548	0.40828

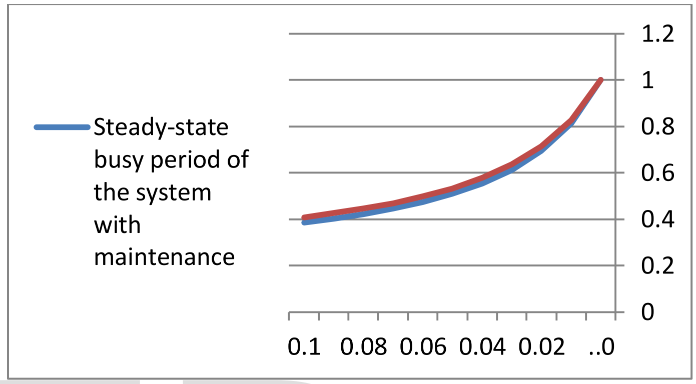

Figure (6):Variation of steady-state busy period with and without maintenance for different values of α_1

f- The expected total profit incurred to the system in the steady-state

If we put: $\alpha_1 = .01, .02, .03, ...$ at constant $\alpha_2 = ,04, \beta = .05, \beta_2 = .06, \lambda = .02, \delta = .02$, $R = 1000, C_1 = 100$, $C_2 = 50$,in Equations (7 – 20), (7 – 21), we get Table 6. Variation of steady state profit w.r.t. failure rate of type I α_1 is shown in Figure 7.

Table (6): Variation of the steady-state profit with respect to failure rate β_1

β_1	Steady-state profit of the system with maintenance	Steady-state profit of the system without maintenance
.0	948.5	948.38
.01	934.76	933.58
.02	921.98	919.89
.03	910.05	907.18
.04	898.9	895.36
.05	888.46	884.33
.06	878.64	874.03
.07	869.42	864.35
.08	860.71	855.29
.09	852.5	846.74
.1	844.73	838.69

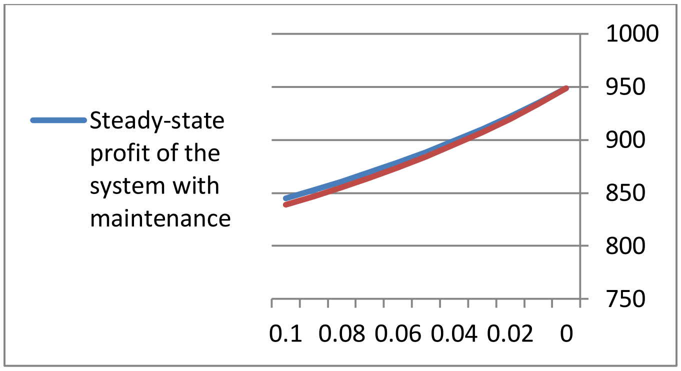

Figure (7): Variation of steady-state profit with and without maintenance for different values of α_1

If we put: $\alpha_1 = .01, .02, .03, \ldots$ at constant $\alpha_2 = ,04, \beta = .05, \beta_2 = .06, \lambda = .02, \delta = .02, R = 1000, C_1 = 100, C_2 = 50$, in Equations (7 – 20), (7 – 21), we get Table 6. Variation of steady state profit w.r.t. failure rate of type I α_1 is shown in Figure 7.

Table (6): Variation of the steady-state profit with respect to repair rate α_1

α_1	Profit of the system with maintenance	Profit of the system without maintenance
.01	654.28	649.13
.02	709.19	702.29
.03	746.83	739.42
.04	774.24	766.80
.05	795.08	787.85
.06	811.48	804.52
.07	824.71	818.06
.08	835.61	829.27
.09	844.75	838.7
.1	852.51	846.74

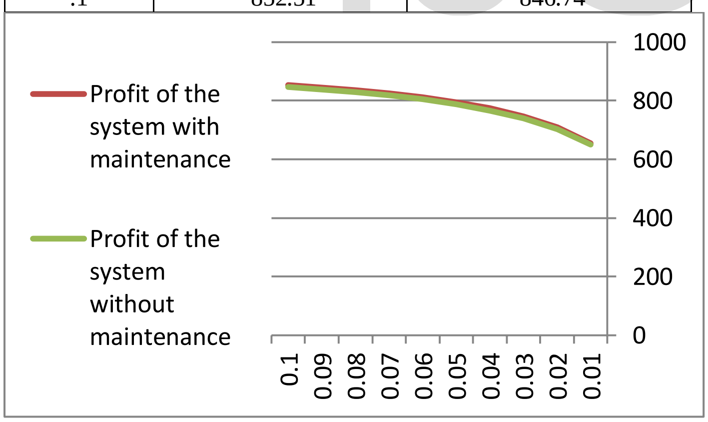

Figure (7): Variation of steady-state profit with and without maintenance for different values of α_1

CONCLUSION

In this study, we developed the explicit expressions for MTSF, availability, steady-state availability and profit function of 3-out-of-4 repairable system and perform comparative analysis Author in Ref. (10) has studied the cost analysis of a system with preventive maintenance by using the Kolmogorov's forward equations method. He has compared the characteristics, MTSF, and the profit function with respect to failure rate α for the system numerically and graphically. He has assumed that the repair rate and preventive maintenance rate are constant.

This paper investigated the system discussed in Ref. [10] incorporating the concept that repair rates and preventive maintenance rate are generally distributed. We analyzed the system by using supplementary variable technique. Variation of the availability and expected total profit w.r.t. time are computed. Also, the MTSF, steady-state availability, and steady-state profit of the system are obtained numerically and graphically.

Table (1) includes the relationship between availability of the system and time and shows that it decreases w.r.t. time and tend to zero after a sufficient long interval of time. Variation of availability w.r.t. time is shown in Figure 2. Table (2): shows expected total profit function in the interval (0, t] for different values of 't'. Figure (3) shows expected total gain increase in the interval (0, t] for first case.

By comparing the characteristic: MTSF, steady state Availability and the steady state profit with respect to failure rate β_1 for the system graphically, it was observed that the increasing of failure rate β_1 at constant α1=0.3, α2=0.4,α3=0.5, α4=0.6, β2=.04,β3=0.05,β4=.06 λ=0.02,, δ=0.2,R=1000,C1=100 ,C2=50, the system characteristic, MTSF, steady state availability and the steady state profit of the system decreased.

Also, by comparing the characteristic, MTSF, steady state Availability and the steady state profit with respect to repair rate α_1 for the system graphically, it was observed that there is an increase of repair rate α_1 at constant α2=0.4, α3=0.5, α4=0.6, β1=.03, β2=0.04, β3=0.05, β4=.06 λ=0.02,, δ=0.2, R=1000, C1=100 ,C2=50, the system characteristic, MTSF, steady state availability and the steady state profit of the system increased.

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